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COMMENTS ON PRESENTATION BY PAUL COX

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As the last of the discussants to give my comments in writing I have the advantage of a preview of what my predecessors have said. I will attempt to summarize and amplify their competent comments.

Mr. Cox has certainly described a problem which has raised many questions of considerable interest. His problem is concerned with the design and analysis of an experiment in which the 'response' is measured in the form of a curve (thrust curve of a motor is his example). Actually Mr. Cox is almost exclusively concerned with analysis. I will later make a few remarks on the design aspect.

Let me start by saying that we cannot really talk about an 'appropriate analysis' of a set of experimental response curves without being clear about

(a) The purpose or the objectives of the experiment

and at least to some extent about

(b) The physical mechanism generating the experimental responses.

With regard to (a) Mr. Cox has described essentially two objectives, namely, the effect of 'Conditioning Temperature' and 'Mix' on the 'Shape of the Thrust Curve' and the 'Total Impulse'. The latter is a single response clearly defined as an integral of the response curve and obtainable (say) by numerical integration. The latter requires clearer definition in terms of thrust curves characteristic of real interest to the engineer and to be specified by him. We may speculate that one of these may be the initial rate at which the thrust increases from zero, or possible the time at which it reaches the stationary stage, etc. Mr. Cox has, however, pointed to an important feature, namely that in general a multiplicity of responses will have to be computed from the curve representing the relevant summaries of interest to the engineer. Following Dr. Lucas's notation and denoting by $y_{ij}(t)$ the thrust for the j^{th} unit of the i^{th} treatment group observed at time t, we would compute for each curve k summaries $S_r(y_{ij}(t))$; $r = 1, 2, \ldots, k$, which in this example, may well be computed from standard formulas of numerical integration and differentiation. To answer the purpose of the experiment we may in many cases apply the well established techniques of multivariate (k-variate) analysis of variance (see, e.g., Smith, H; Gnanadesikan, R. and Hughes, J. B. (1962)) to the $S_r(y_{ij}(t))$ which would be a 3×3 factorial (i) by temperature and mix with 3 replicate units (j) in each cell. Much of the information will often be obtainable from a standard single variate analysis of variance applied to each of the $S_r(y_{ij}(t))$ separately. This sort of analysis which uses a 'between unit error' is also recommended by Dr. Lucas and Dr. Federer, called 'robust' by the former, and is in essence identical with Mr. Cox's analysis of variance for the regression intercept, a, and slope, b, fitted to a 'straightlooking' section of the curve. I question, however, whether Mr. Cox's procedure of 'arbitrarily' breaking up the curve into sections and fitting polynomials separately to the sections really contributes to our appreciation of the engineering aspects Is it really of interest to the engineer that a cubic term in the first section goes up with temperature? To my mind it is of the greatest importance to communicate with the engineer on the selection of relevant summaries $S_r(y_{ij}(t))$.

This brings me to (b), namely, the importance of a physical theory leading to a mathematical model for the thrust $y_{ij}(t)$, stressed by all discussants. Dr. Lucas postulates a model of the form

$$(1) \qquad y_{ij}(t) = \phi(t;\ \theta_{ij}) + \epsilon_{ij}(t)$$

where θ_{ij} is a (say) m vector of parameters. Whilst in the present example it should be quite feasible to obtain such a model from (say) the differential equations governing the dynamics of the thrust phenomenon, the statistician may be called upon to analyze curves arising in a situation in which the setting up of a mathematical model is difficult. I would stress, therefore, that summaries $S_r(y_{ij}(t))$ answering the purpose of the experiment can often be decided upon <u>without</u> reference to a mathematical model, although the study of their statistical efficiency is facilitated by the mode. Where the latter is available one may proceed as Dr. Lucas suggests to estimate the 'treatment averages' θ_i^* of the θ_{ij} although it may be argued to be more appropriate to estimate the treatment averages of the relevant summaries $S_r(\phi\ (t;\ \theta_{ij})$ the two being differentially equivalent.

Whatever method is used I believe that <u>some</u> attention <u>should</u> be given to the estimation of the individual units', θ_{ij}, and this raises the question of the 'within curve' error or noise. I agree with Dr. Lucas that this will often be relatively unimportant. However, it is of the same degree of relevance as, for example, the estimation of the mean life for a time mortality curve or the L. D. 50 of a dosage mortality situation both of which are usually 'within curve' estimation problems and both have received considerable (possible exaggerated) attention by statisticians. I will, therefore, answer the question raised by Mr. Cox concerning the within curve error structure: --The 'degrees of freedom' that are to be attached to a set of risiduals $y_{ij}(t_s) - \phi(t_s, \theta_{ij})$ computed at some arbitrarily selected time points, t_s, appear to depend on the choice of the t_s. Because of the time series correlogram the sum of squares of residuals

$$\sum_{s=1}^{s} (y_{ij}(t_s) - \phi(t_s, \theta_{ij}))^2$$

is approximately distributed as $c\chi^2$ based on an 'effective number' of degrees of freedom (see, e.g., Bayley and Hammersley (1946)) and ideally this should be invariant with the choice of grid points, t_s. Without the knowledge of the correlogram one cannot judge how the degrees of freedom of Mr. Cox's Figure 4 are affected. However, the analysis given in this figure in any case does not take proper account of the distinction between 'within curve' and 'between curve errors' as was pointed out by Dr. Lucas.

Finally a few words on the question of the design of an experiment with curve responses. First let me say that the choice of t_s values is not a question of the design (as Dr. Lucas rightly stresses but as far as I recall nobody said so during the discussion). This is a computational question of analysis. The design is concerned with the choice of the levels of 'temperature' and the composition of the mixes or, indeed, with questions of what treatment combinations should be chosen. Since the work by Box and Draper (1959) and Kiefer (1959) and others is mainly concerned with a single experimental response or a single response surface, much needs to be done about designing experiments which in some sense are optimum for the 'assessment' of multiple response surfaces, particularly if (as is the case with Mr. Cox's example) some factors (mix) are qualitative. In the absence of a comprehensive theory one would perhaps single out one important response from the $S_r(y_{ij}(t))$ and optimize the design for it using some such theory as the above but optimizing subject to tolerances for the bias and precision with which reponse surfaces for the other response surfaces can be estimated.

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